Dissipation of magnetic fields in neutron star crusts due to development of a tearing mode

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Abstract

Dissipation of magnetic fields in Hall plasma of neutron star crusts may power persistent high energy emission of a class of strongly magnetized neutrons stars, magnetars. We consider development of a dissipative tearing mode in Hall plasma (electron MHD) and find that its growth rate increases with the wave number of perturbations, reaching a maximum value intermediate between resistive τ_r and Hall times τ_H, $\Gamma \sim 1/\sqrt{\tau_r \tau_H}$. We argue that the tearing mode may be the principal mechanism by which strong magnetic fields are dissipated in magnetars on times scale of $\sim 10^4 - 10^5$ yrs powering the persistent X-ray emisison.

1 Introduction

Evolution of neutron stars' magnetic fields is one of the principal issues in neutron star research. On the one hand, observationally, there is no evidence that fields decay on long time scales, of the order of million years [1]. Yet, this constrains the dipolar component of a magnetic field, generated, presumably, by currents flowing in the superconductive core. The fate of a magnetic field in the crust may be different. It is tempting to relate an X-ray activity of magnetars, strongly magnetized neutron stars, to the decay of a crustal magnetic field created by a dynamo action during the neutron star birth [2, 3].

One of the central problems is how fast a magnetic field can be dissipated. Conventionally, the magnetic field decay time scale is assumed to be of the order of the resistive time scale, given by the size of the system L (of the current-carrying layer, to be more precise) and resistivity η, $\tau_r \sim L^2/\eta$. In fact, plasma has a number of ways to dissipate magnetic field on time scales much shorter than τ_r (otherwise we would never see a Solar flare, since the resistive time scale in the Solar corona is longer than the age of the Universe). Typically, fast magnetic field dissipation is achieved through formation of small scale structures, with correspondingly short dissipation time scales. One possibility to create small scale structures is through non-linear interaction of waves, either locally (in phase space) through formation of a Hall cascade [4], or non-locally due to large scale motions leading to "wave overturn" [5, 6, 7, 8, 9].

Here we discuss another way to form small scale dissipative current layers via development of a tearing mode in Hall plasma. In Hall plasma (sometimes called electron MHD, EMHD below [5]) ions are assumed to be motionless, providing a neutralizing background for electron fluid. Tearing mode [10] is the principal resistive instability of current-currying plasma and is one of the key factors leading to an explosive release of magnetic energy in Solar flares [11] and magneto-tail [12]. It is also important for TOKAMAK discharges like sawtooth oscillations and major disruptions [13]. Tearing mode in regime is an important ingredient of modern reconnection models [7, 14]. During the development of the tearing mode, perturbations of a plasma equilibrium lead to formation of current sheets where both the time scales for diffusion may be short and, in addition, resistivity may be enhanced due to the development of plasma turbulence (anomalous resistivity). A current layer tends to be locally unstable to transverse, $\mathbf{k} \cdot \mathbf{B}_0 = 0$, perturbations ($\mathbf{k}$ is the wave vector of perturbations and $\mathbf{B}_0$ is initial magnetic field). Qualitatively, divergence at points $\mathbf{k} \cdot \mathbf{B}_0 = 0$ is related to the fact that Alfvén velocity becomes zero at these points, so that perturbations are effectively piling up. Since whistler waves, normal modes in Hall plasma, also have phase speed equal to zero at points where $\mathbf{k} \cdot \mathbf{B}_0 = 0$, we might expect a somewhat similar behavior. This is indeed what we show below.

2 Tearing mode in Hall plasma

2.1 Tearing due to electron resistivity

In deriving growth rates for the tearing mode in Hall plasma we follow a standard scheme of electron-ion plasma [15]. On scales much larger that electron skin depth, the equation of resistive EMHD is (for example [5])

$$\partial_t \mathbf{B} = \eta \Delta \mathbf{B} - \frac{c}{4\pi e} \nabla \times \left(\frac{\nabla \times \mathbf{B}}{n} \times \mathbf{B} \right) \tag{1}$$

where η is resistivity, n is plasma density and other notations are standard. In what follows we assume density to be constant.

We assume that initial magnetic field configuration is stable both on Alfvén time scale (implying a dynamical stability) and on Hall time scale. In doing so we wish to single out the effects of the tearing mode as opposed to dynamical instabilities, Hall cascade etc.

Let the initial magnetic field correspond to current layer $\mathbf{B} = B_0(f_x(z)\mathbf{e}_x + f_y(z)\mathbf{e}_y)$ with $f_x \to \pm 1$ at $z \to \pm\infty$ and $f_x = 0$ in the center of the layer $z = 0$. (If the layer is force-free, then $f_x^2 + f_y^2 = 1$; this is **not** required for tearing instability). Next we impose perturbation of the field in the form

$$\delta \mathbf{B} = \nabla \times \xi \tag{2}$$

where $\xi \propto e^{-i(\omega t - k_x x - k_y y)}$ is vector potential. We foresee instability at points $\mathbf{k} \cdot \mathbf{B}_0 = 0$, so to simplify mathematics, we assume $k_y = 0$, so that a current

sheet will be created at $z=0$. The equations for evolution of perturbations are

$$
\begin{aligned}
&\omega\xi_y' + \frac{c^2\mathbf{k}_x\omega_B}{\omega_p^2}\left(ik_x(f_x'\xi_z + f_x\xi_z') - \xi_y f_y'' - \xi_x'' f_x - f_x'\xi_x' - f_y'\xi_y'\right) + \\
&\eta\left(i(k_x^2\xi_y' - \xi_y^{(3)}) - \frac{cm\omega_B}{e}f_x''\right) = 0 \\
&\omega(ik_x\xi_z - \xi_x') + \frac{c^2\mathbf{k}_x\omega_B}{\omega_p^2}\left(k_x^2 f_x\xi_y + \xi_y f_x'' - f_x\xi_y''\right) \\
&-\eta\left(k_x(k_x^2 + \xi_z'') + \frac{cm\omega_B}{e}f_y'' + i(k_x^2\xi_x' - \xi_x^{(3)})\right) = 0 \\
&-ik_x\omega\xi_y + \frac{c^2\mathbf{k}_x^2\omega_B}{\omega_p^2}\left(k_x f_x\xi_z + i\xi_y f_y' + if_x\xi_x'\right) + \eta k_x\left(k_x^2\xi_y - \xi_y''\right) = 0 \quad (3)
\end{aligned}
$$

The two scales of interest that appear in the equations are

$$
\begin{aligned}
\delta &= \sqrt{\frac{\eta}{|\omega|}} \\
\delta^* &= \sqrt{\frac{L|\omega|\delta^2\omega_p^2}{c^2k_x\omega_B}} = (Lk_x\delta^2)\frac{|\omega|}{\omega_w}
\end{aligned} \tag{4}
$$

where $\omega_w = (ck_x)^2\omega_B/\omega_p^2$ is the frequency of whistler waves. Generally, dispersion relation of whistler waves $\omega_w = c^2kk_x\omega_B/\omega_p^2 = c^2k(\mathbf{k}\cdot\omega_{\mathbf{B}})/\omega_p^2$, where $k = \sqrt{k_x^2 + k_z^2}$. In an inhomogeneous plasma of our choice $\omega_w = 0$ at a point $z = 0$. For comparison, in e-i plasma $\delta^* = \sqrt{L\delta|\omega|/\omega_A}$, with $\omega_A = k_x v_A$. In the above equations $|\omega|$ is an absolution value of the (generally complex) frequency ω. As it turns out, real part of ω is not important (see below), so $|\omega| \sim \Gamma$, where Γ is a growth rate of instability.

A standard method in describing the evolution of tearing mode is similar to the boundary layer problem. It involves separation of a current layer into "bulk", where derivatives are small and resistivity is not important, and a narrow "boundary layer" where derivatives and resistivity may be large. Two different approximations are done in each layer - ideal and weakly varying plasma in the bulk and a narrow resistive sublayer, with two solutions matched continuously.

In the outside region of smooth field evolution, where we can neglect resistivity, the system (3) reduces to

$$
\xi_y'' - \left(k_x^2 - \frac{\omega^2\omega_p^4}{c^4\omega_B^2k_x^2f_x^2} - \frac{\omega\omega_p^2f_y'}{c^2k_x\omega_Bf_x^2} + \frac{f_x''}{f_x}\right)\xi_y \tag{5}
$$

We are interested in the slow evolution of plasma and thus neglect terms proportional to ω. The structure of magnetic field in the ideal region is then described by

$$
\xi_y'' = \left(k_x^2 + \frac{f_x''}{f_x}\right)\xi_y \tag{6}
$$

This clearly shows that instability is driven by f''_x term, the second derivative of magnetic field.

The simplest choice of the background field in (6) is $f_x = \sin z/L$, $z < L$, in which case

$$\xi''_y = \left(k_x^2 - \frac{1}{L^2}\right)\xi_y \tag{7}$$

with solution

$$\xi_y = C_1 \cos\left(\frac{\sqrt{1-k_x^2L^2}z}{L}\right) + C_2 \sin\left(\frac{\sqrt{1-k_x^2L^2}z}{L}\right) \tag{8}$$

Outside of the current sheet, $z > L$, $\xi_y = e^{-k_x z}$ and the matching gives

$$\begin{aligned} C_1 &= e^{-k_x L}\left(\cos\sqrt{1-k_x^2L^2} + \frac{k_x L \sin\sqrt{1-k_x^2L^2}}{\sqrt{1-k_x^2L^2}}\right) \\ C_2 &= e^{-k_x L}\left(-\sin\sqrt{1-k_x^2L^2} + \frac{k_x L \sin\sqrt{1-k_x^2L^2}}{\sqrt{1-k_x^2L^2}}\right) \end{aligned} \tag{9}$$

Thus, magnetic field $B_z = k_x \xi_y$ is continuous at $z = 0$, but its derivative experiences a jump

$$\Delta = [\ln B_z] \sim \frac{1}{L} \tag{10}$$

For instability it is required that $\Delta > 0$. (Recall, for comparison, that in electron-ion plasma $\Delta \sim 1/(kL^2)$).

In the resistive sub-layer, at $z \ll L$, we have $f_x \to 0\,, f'_y \to 0$, so that the z component of (3) gives

$$\xi''_y = \frac{1 + k_x^2\delta^2}{\delta^2}\xi_y \tag{11}$$

where $\delta^2 = \eta/\Gamma$, $\Gamma = \mathrm{Im}(\omega)$. In the limit $k_x\delta \ll 1$ this gives

$$\xi_y = \cosh z/\delta \tag{12}$$

where a boundary condition $\xi'_y = 0$ at $z = 0$ was used. (Structure of the resistive sub-layer is the same as in electron-ion plasma). The inner and outer solutions match at

$$\delta^* = \Delta\delta^2 \tag{13}$$

where δ^* is the thickness of the resistive sub-layer.

From Eqns (4), (13) we get the growth rate

$$\Gamma = \frac{c^2 k_x \delta^2 \Delta^2 \omega_B}{L\omega_p^2} = \left(\frac{c^2 k_x \eta \omega_B}{L^3 \omega_p^2}\right)^{1/2} \tag{14}$$

This is consistent with Eq. 50 in Ref. [7], and Eq. (68) in Ref. [16]; see also [17, 18]. If we define phase velocity of whistler modes as $v_w = k_x c^2 \omega_B/\omega_p^2$, then Eq. (14) takes the form

$$\Gamma = \frac{1}{\sqrt{\tau_r \tau_w}} \tag{15}$$

where resistive time scale is $\tau_r = L^2/\eta$ and whistler time scale $\tau_w = L/v_w$.

Growth rate (14) increases as $k_x^{1/2}$. This is in stark contrast to electron-ion plasma, where growth rate decreases with k_x (as $\propto k^{-2/5}$, *e.g.* [15] Eq. 7.123). Thus, in Hall plasma the maximum growth rate is reached at $k_x \sim 1/L$, and becomes

$$\Gamma = \frac{c\sqrt{\eta\omega_B}}{L^2\omega_p} = \frac{1}{\sqrt{\tau_r\tau_H}} \tag{16}$$

where we defined Hall time

$$\tau_H = \frac{L^2\omega_p^2}{c^2\omega_B} \tag{17}$$

Relation (14) gives the growth rate of tearing mode in Hall plasma. Thus, the time scale for development of a tearing mode is intermediate between the resistive and Hall time scales, and thus is much shorter than resistive time scale. This expression reminds of the maximum growth rate of tearing mode in electron-ion plasma, which is intermediate between resistive and Alfvén time scales.

Growth rate (16) may also be written as

$$\Gamma = \left(\frac{c}{L\omega_p}\right)^2 \frac{\omega_B}{\sqrt{S}} \tag{18}$$

where we introduced an effective Lundquist number associated with Hall time $S = v_H L/\eta$, $v_H = L/\tau_H$

2.2 Tearing due to inertial resistivity

In a highly conducting plasma, electron inertia may play a role of resistivity, providing a relation between the electric field and the current. It is this regime that most works on tearing mode in Hall plasma addressed so far [7]. Since in this case the width of the current sheet may become comparable to electron skin depth, one has to take into account additional terms in the Ohm's law and it's consequence, Eq. (1) If we neglect this possibility (this is justified since we get correct expression for the growth rate, see below), the effective resistivity in the collisionless regime becomes $\eta_{eff} = \Gamma\,(c/\omega_p)^2$ [19, 7]. Thus, inertial resistivity dominates when $\Gamma(c/\omega_p)^2 > \eta$, which can be written as $(c/L\omega_p)^2\sqrt{\tau_r/\tau_w}$.

The corresponding tearing mode growth rate is

$$\Gamma = (k_x L)\left(\frac{c}{L\omega_p}\right)^4 \omega_B = \Delta'^2 \frac{c^4 k_x \omega_B}{L\omega_p^4} \tag{19}$$

This is the same as the growth for electron tearing mode cited in Ref. [20]. The maximum rate is reached at $k \sim 1/L$

$$\Gamma_{max} = \left(\frac{c}{L\omega_p}\right)^4 \omega_B \tag{20}$$

The ratio of resistive (16) and inertial (20) maximum growth rates is $(c/L\omega_p)^2\sqrt{S}$.

3 Application to neutron stars

High energy emission of strongly magnetized neutron stars (magnetars) is powered by dissipation of magnetic field [3, 21, 22]. Strong magnetic fields, of the order of 10^{14} G, may be created by a dynamo mechanism, *e.g.* of the $\alpha-\Omega$ type, operating at birth of neutron stars [2]. After ~ 100 secs from the birth, the crust of the neutron star solidifies. This time is much longer than Alfvén crossing time $\sim 0.1-1$ s, so that after the end of the turbulent motion and before the crust formation, magnetic field in a star should evolve to some minimum energy state allowed by the system.

After the crust solidifies, the evolution of magnetic field will proceed due to non-dissipative Hall effect, Ohmic resistivity, and, as we argue in this paper, due to development of tearing instability. In order to estimate the corresponding times scales, we use resistivity $\eta = c^2/4\pi\sigma$ with conductivity $\sigma \sim 10^{25}\rho_{12}^{2/3}$ [23] evaluated at the neutron drip point $\rho \sim 10^{12}$ g/cm^{-3}. The Hall τ_H, resistive τ_r and tearing time scales (given by inverse of the tearing mode growth rate (14)) become

$$\begin{aligned}
\tau_H &= 4\times 10^3 L_4^2 B_{14}^{-1}\rho_{12}\,\text{yrs} \\
\tau_r &= 4\times 10^5 L_4^2 \rho_{12}^{2/3}\,\text{yrs} \\
1/\Gamma &= 4\times 10^4 L_4^2 \rho_{12}^{5/6} B_{14}^{-1/2}\,\text{yrs}
\end{aligned} \tag{21}$$

where a standard subscript notation, *e.g.* $L_4 = (L/10^4\text{cm})$, has been adopted. This indicates that growth rate of the tearing mode in Hall plasma is of the same order as the activity time of magnetars. Also, for higher magnetic field the instability time is shorter while corresponding magnetic energy is larger, consistent with the fact that only high field neutron stars exhibit magnetar-like activity. One can also check that in neutron star crusts resistive effects well dominate over inertial ones.

We envision a magnetar scenario similar to the ones proposed in Refs [24, 25]. After the turbulence seizes, initially stable magnetic field configuration forms which is then dissipated due to development of tearing mode on time scale $\sim 10^4 - 10^5$ yrs. This leads to Lorentz force disbalance in the crust, which initially can be compensated by crust tensile strength, but eventually is released through crust deformation leading to bursts and flares. (Release of crustal stress can be either through cracks [2] or through plastic deformations [26]).

Since small scale currents are dissipated on shorter time scales, while larger currents take longer to dissipate, we expect that *giant flares are more common in older magnetars.* (This is also a natural consequence of the torus formation [27], in which case small scale magnetic fields are dissipated soon after the birth of a neutron star). This may provide a resolution to the energy budget problem for magnetar giant flares: the giant flare of SGR 1806 - 20 emitted 2×10^{46} ergs in high energy γ-rays [28], and associated mechanical energy is expected to be even larger. If a typical giant flare recurrence time is ~ 30 yrs (the period of active monitoring of high energy sky) and activity time is several thousand

years (characteristic age) the magnetic field, which powers the flares, has to be *larger* than 3×10^{15} G, which is on the verge of being uncomfortably large.

4 Conclusion

In this paper we first considered development of a resistive tearing mode in Hall plasma. Tearing mode develops on time scale intermediate between Hall and resistive time scales. Qualitatively, this might have been expected in analogy with the tearing mode in electro-ion plasma. Unlike the case of electron-ion plasma, the growth rate (14) increases with k, reaching maximum for $k \sim 1/L$, the thickness of the current layer. This is due, qualitatively, to the dispersion of whistler modes, for which phase velocity increases with k.

We then argue that the development of the tearing mode in the crust of strongly magnetized neutron stars, magnetars, powers their high energy emission on time scales $\sim 10^4 - 10^5$ years. One of the principal uncertainty is the length scale L of the current sheets created by the dynamo, as all the time scales are strongly dependent on it, Eq. (21). For our choice of parameters the Hall time scale is $\tau_H \sim 10^4$ yrs. According to Ref. [23], this, in fact, applies to a very broad density range, $\rho \sim 10^{11} - 10^{13}$ g cm^{-3}. Ohmic decay times are strongly dependent on the temperature, with the value that we used, $\tau_R \sim 10^6$ yrs., giving a reasonable approximation.

Overall, the evolution of a crustal magnetic field is bound to be a combination of a dissipative tearing mode and a nondissipative Hall cascade. The Hall turbulent cascade can create small scale structures which in turn become dissipative [4]. The time it takes for a cascade to propagate down to dissipative scale is, typically the large eddy overturn time times the logarithm of the ratio of outer to inner scales, $\sim \tau_H \ln L/L_{in} \gg \tau_H$ [29] , where L_{in} is the dissipative (resistive) scale. As the ratio L/L_{in} is large, it takes many Hall times to dissipate magnetic energy. Numerical simulations [30] indeed seem to indicate that the transfer of energy to the higher harmonics is not sufficient to accelerate significantly the decay of the original field.

Relative importance of Hall cascade and tearing mode depend on details of the structure of magnetic field before solidification of the crust. This is yet an unresolved issue, as is exemplified by a long standing problem of stability of magnetic field in stars [31, 32, 27, 33, 34]. It appears that evolution at intermediate times (longer than Alfvén crossing time but shorter than dissipative times) depends on magnetic helicity, either at the end of the turbulent phase or due to helicity imbalance due to loss through the surface). For certain parameters, internal magnetic field relaxes to a complicated torus-like form with comparable toroidal and poloidal magnetic fields.

Our results provides an elegant explanation to observation in Ref. [35] (see also [36]) where it is found that formation of dissipative current sheets requires at least quadratic dependence of the background field on the position. Eq. (6) clearly shows that instability is driven by the second spacial derivative of the field. Thus, what is calculated in Refs [35] is a tearing mode. Finally, in

Ref. [8] a somewhat related problem was considered, the formation of a current sheet in presence of a steep density gradient. In contrast, the tearing mode considered in this paper does not require density gradient, all that is needed is an inhomogeneous magnetic field. In addition, limitation of purely toroidal field, assumed in [8] seems to overestimate the dissipation [37].

Finally, the model presented here may be directly probed in the laboratory, in particular at the field-reversed configuration experiment at UCLA [38]. Tearing and formation of magnetic islands, a typical consequence of the nonlinear development of the tearing mode, are clearly seen (*e.g.* Fig 9 in Ref [38]).

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